Conceptual Understanding and Procedural Fluency in Mathematics – Some Examples

Both procedural fluency and conceptual understanding are necessary components of mathematical proficiency and mathematical literacy. In support of problem solving, teachers, students, and parents should work to develop both.

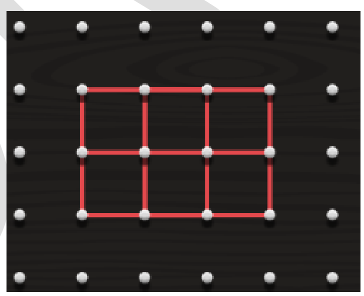
Example 1: Compute 3.14×4.5

Conceptual Understanding: *comprehension of mathematical concepts, operations, and relations*	**Procedural Fluency:** *the ability to apply procedures accurately, efficiently, and flexibly*
Students have an understanding that the product 3.14×4.5 must be between 12 and 20 and might more accurately estimate the product to be close to 14	Students use a procedure or algorithm to compute the product $3.14 \times 4.5 = 14.13$, and they use their *conceptual understanding*, to assess the reasonableness of the result

Example 2: What is 25% of $84?

Conceptual Understanding: *comprehension of mathematical concepts, operations, and relations*	**Procedural Fluency:** *the ability to apply procedures accurately, efficiently, and flexibly*
A student with strong conceptual understanding might recall that 25% is the same as ¼ , and be able to use mental math to find the result, understanding that "25% of", "one-fourth of", multiplying by 0.25, and dividing by 4, are all different ways to understand this one problem.	A student with procedural knowledge and fluency could generalize their conceptual knowledge to the procedure of finding the product of 0.25 x 84 = 21 to solve the problem. This procedure could be practiced to improve the efficiency of solving this common type of problem.

Example 3: Understanding Area



Conceptual Understanding: *comprehension of mathematical concepts, operations, and relations*	**Procedural Fluency:** *the ability to apply procedures accurately, efficiently, and flexibly*
Students can understand conceptually that the two by three rectangle above has an area of 6 square units, without the use of an area formula or algorithm. The idea or *concept* that area is measured in square units is supported by the visual.	Students could recognize the multiplicative relationship between the side lengths and the area and use this observation to identify a procedure for efficiently computing the area of any rectangle $(length \times width)$.

- Conceptual Understanding can be defined as the "comprehension of mathematical concepts, operations, and relations" (National Research Council, 2001)

- The National Council of Teachers of Mathematics (NCTM - USA) recommends eight effective math teaching practices that "research indicates need to be consistent components of every mathematics lesson" (NCTM, 2014). These practices include a call to build procedural fluency from conceptual understanding, stating that "Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skilful in using procedures flexibly as they solve contextual and mathematical problems."

- Effective mathematics teaching focuses on the development of *both* conceptual understanding *and* procedural fluency. Major reports have identified the importance of an integrated and balanced development of concepts and procedures in learning mathematics (National Mathematics Advisory Panel 2008; National Research Council 2001). Furthermore, NCTM (1989, 2001) emphasizes that procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving.

- To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results. Martin (2009, p. 165)

References:

National Council of Teachers of Mathematics (NCTM), 2001. *Principles for School Mathematics*

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National Research Council, 2001. *Adding It Up*

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