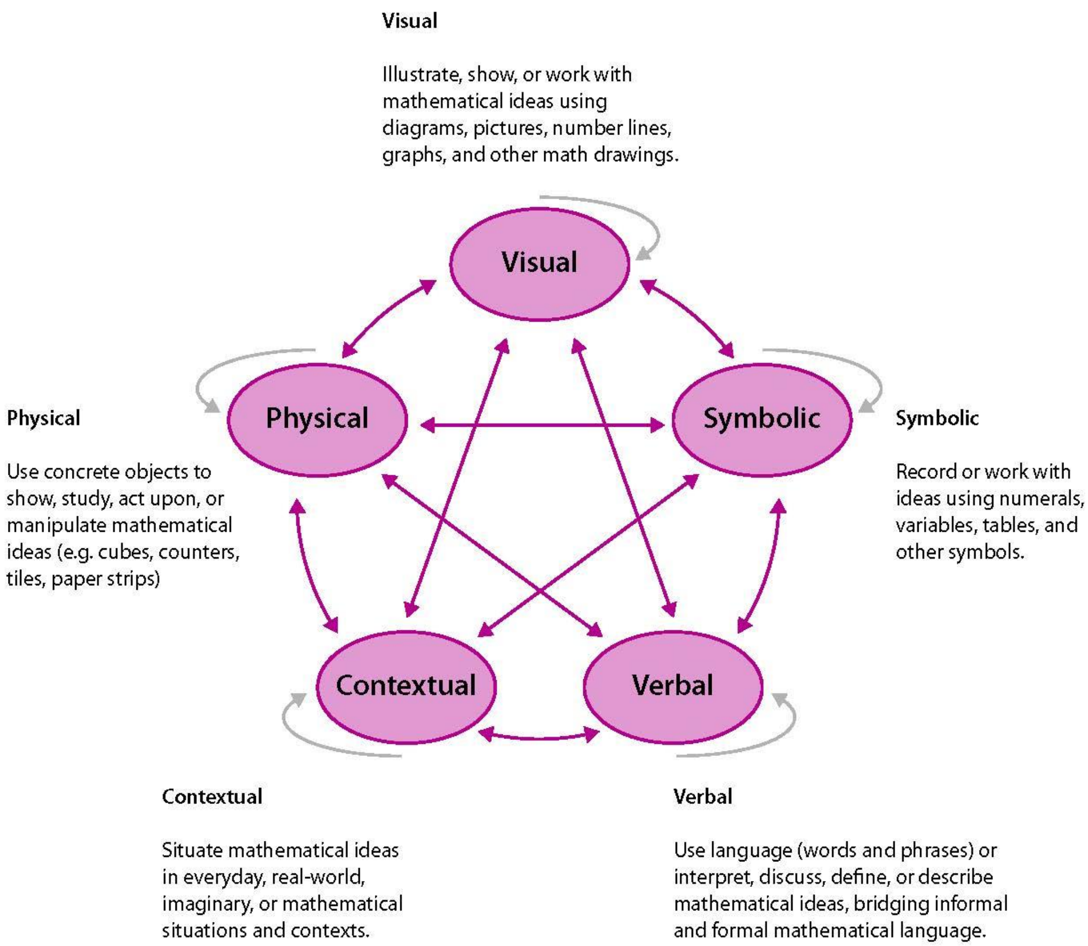
Mathematics | Use & Connect Mathematical Representations

The CBE K-12 Mathematics Framework is a key document that outlines the *how* of teaching and learning mathematics in CBE. It explains teaching practices that are essential to effective mathematics education, which includes empowering students with the skills to make sense of mathematics. By providing opportunities that allow learners to see mathematics as a creative and beautiful discipline, value is placed on the process of understanding it rather than simply producing the right answer. One mathematics teaching practice is to *Use and Connect Mathematical Representations*. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen their understanding of mathematics concepts and procedures as well as using representations as tools for problem-solving.



Mathematics | Use & Connect Multiple Representations

To highlight the importance of understanding multiple representations for a mathematical idea, a story is included below:

"Once upon a time, five people who had never seen an elephant were blindfolded and led to an elephant and asked to describe what they thought the creature looked like. The people were allowed to touch the animal. They approached it from different directions.

One felt the tail, the second found its leg, the third reached for the trunk, another felt its ear, and the fifth person touched the great stomach of the animal. Each one then described [their] version of the animal and each was, in turn, completely surprised at the contrasting view offered by the others in the group.

Finally, the blindfolds were removed, and the five looked at the animal that each had described. Each person realized that not only was [their] perspective valid, albeit in a limited way, but also that this applied equally to every member of the group. Only after opening their eyes (literally and metaphorically) and seeing the animal from a variety of angles could they appreciate the complete picture." (Tripathi, 2008, p. 438)

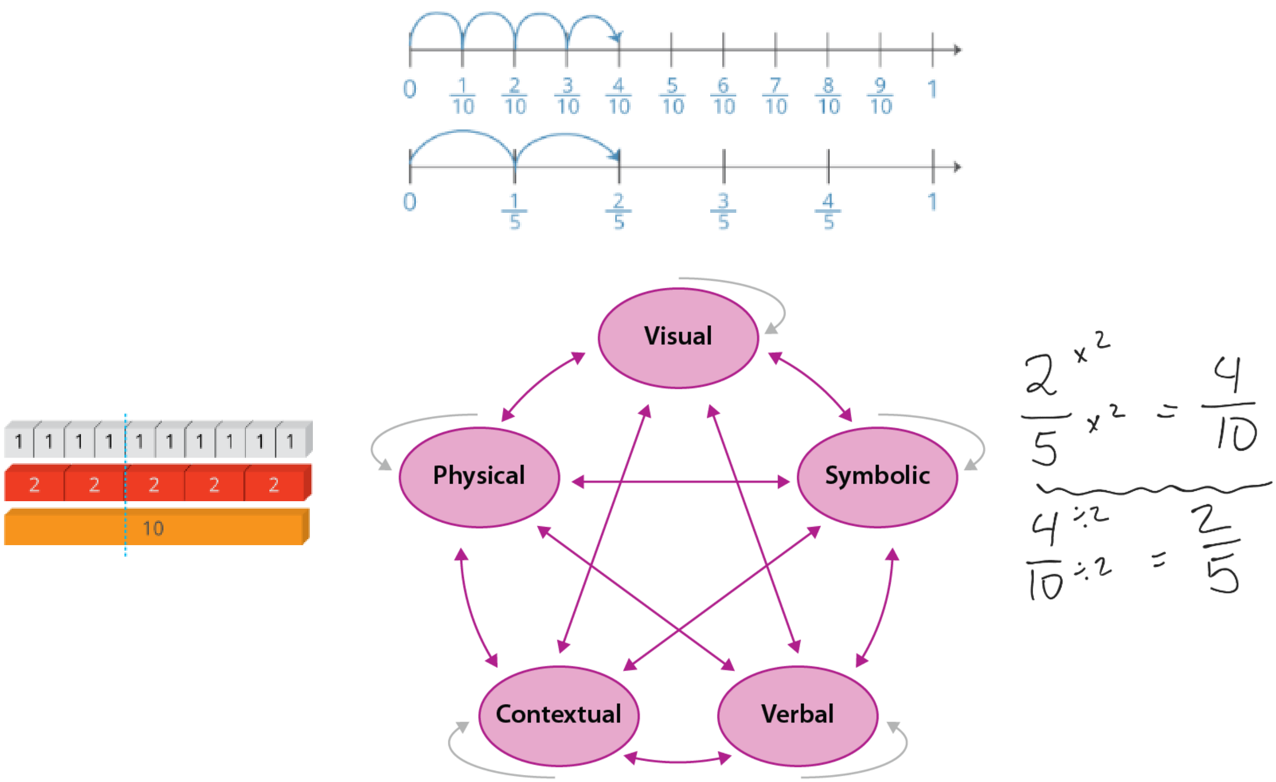
Tripathi (2008) uses this story to show that the use of multiple mathematical representations is critical in supporting students' understanding of mathematical ideas and deepening conceptual understanding.

The world of mathematics is our metaphorical elephant in the classroom waiting for our blindfolds to be removed and explored in multiple ways. Imagine the overwhelming emotion of seeing all the parts of the elephant you didn't realize were there, thinking you knew exactly what you had in front of you. If we only use one representation to approach a mathematical concept or procedure, it is like we approach it blindfolded. If we remove the blindfolds and use multiple representations, we can engage with the concept in a more holistic manner.

Elementary

Overarching Problem

Determine if two fractions are equivalent.



One person says they are $\frac{4}{10}$ km away from the school and another student says they are $\frac{2}{5}$ km away from the school. Both people are the same distance from the school.

"I know two fractions are equivalent if they represent the same quantity.

Equivalent fractions would be in the same position on the number line. $\frac{2}{5}$ and $\frac{4}{10}$ are both the same distance from 0.

I can multiply or divide both the numerator and denominator by the same amount. For example, $\frac{2}{5}$ is equivalent to $\frac{4}{10}$ because 2×2 is 4 and 5×2 is 10."

Note | The task has been used and adapted from MathUP Classroom Grade 5 (n.d.).

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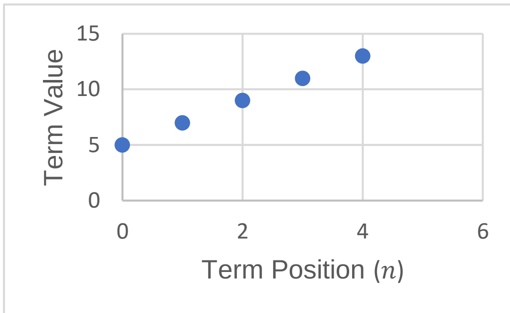
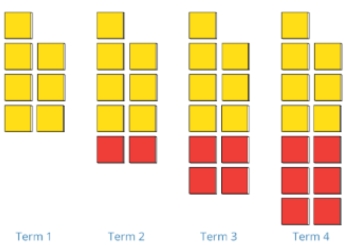
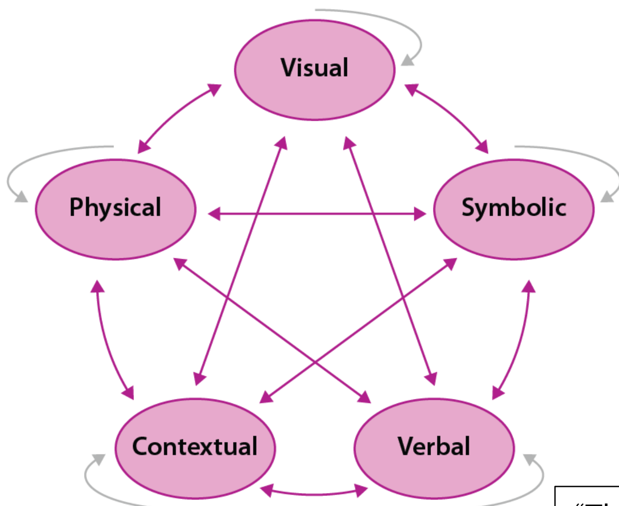
Middle Years

Overarching Problem

Create a pattern that starts with a number and increases by 2.







Term position	Term value
1	7
2	9
3	11
4	13

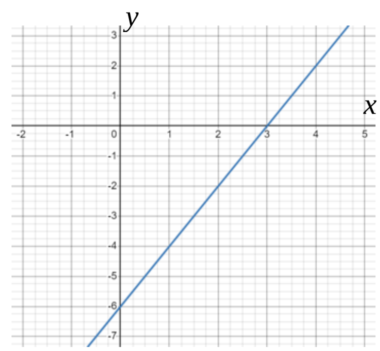
A taxi charges a flat rate of \$5 at the beginning of every fare then \$2 for every kilometer driven.

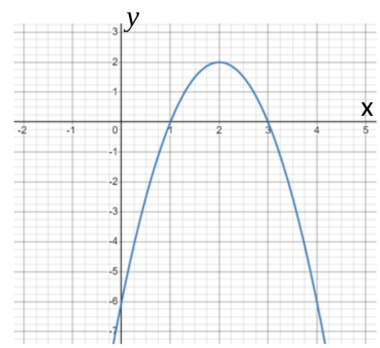
"The term values goes up by the same amount each time, so this is a linear relationship. Because the term value goes up by 2 each time, I know the coefficient of the equation is 2. The y-intercept on the graph is 5. This is the constant. So, I know the linear equation $y = 2n + 5$ represents my pattern."

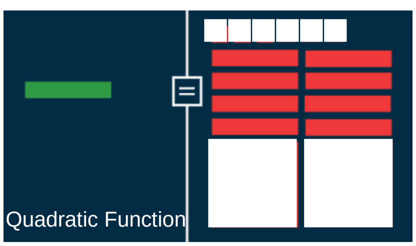
Note | The task has been used and adapted from MathUP Classroom Grade 8 (n.d.).

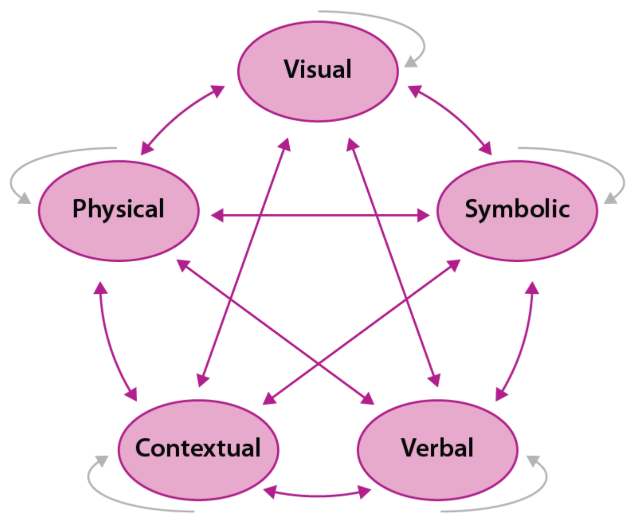
High School

Overarching Question
How is a quadratic function the similar to a linear function? How are they different?









Linear Function
$f(x) = 2x - 6$

Quadratic Function
$f(x) = -2x^2 + 8x - 6$

The linear function may model situations that have a constant rate of change, such as beginning 4 km east of home and moving at a constant rate of 2 km/h west.

A quadratic equation may model situations, such as projectile motion under the force of gravity. For example, hitting a golf ball that is 1 yard away from a pathway and after you hit it, the ball lands on the ground 3 yards away.

In both cases, there would be domain and range restrictions, as for example, the domain would be limited to real numbers greater than or equal to zero.

A linear function has a constant slope, in this case 2, whereas the quadratic function does not have a constant slope.

The linear and quadratic functions both have a y - intercept at $(0, -6)$.

Both functions have the same domain, spanning all real numbers. However, each function has a different range, as the quadratic function reaches a maximum value of 2, whereas the linear function does not have a maximum value.

The linear equation has one x - intercept but the quadratic equation has two x - intercepts.

The linear function has a positive leading coefficient while and the quadratic function has a negative leading coefficient. This impacts the end behaviour of both graphs.

References

CBE. (2022). K-12 Mathematics Framework. https://cbe.ab.ca/about-us/policies-and-regulations/Documents/Mathematics-Framework.pdf

MathUP Classroom. (n.d.). Grade 5: Exploring equivalent fractions.

MathUP Classroom. (n.d.). Grade 8: Describing linear patterns using rules.

Tripathi, P. (2008) Developing mathematical understanding through multiple representations. *Mathematics Teaching in the Middle School*, *13*(8), 438-445.